JUNE 2010

3(a)				
	P x y z s t *valu e*			
	1 –6 –5 –3 0 0 0	M1		Two slack variables used correctly
	0 (1) 2 k 1 0 8	A1		1 row correct
	0 2 10 1 0 1 17	A1	3	all correct
				May earn in (b)(i)
(b)(i)	Pivot in x-column = 1	B1		May be implied by second row unchanged
	1 0 7 $6k$–3 6 0 48	M1		row operations (even with wrong pivot)
	0 1 2 k 1 0 8	A1		1st or 3rd row correct
	0 0 6 1–2k –2 1 1	A1	4	all correct
(ii)	$6k-3<0$	M1		"their" $6k-3<0$
	$\Rightarrow k<\frac{1}{2}$	A1	2	
(c)				
	1 0 7 –9 6 0 48			
	0 1 2 –1 1 0 8			
	0 0 6 (3) –2 1 1	M1		new pivot correct from their tableau and row operations attempted
	1 0 25 0 0 3 51	A1		2 rows correct (may be multiples of rows) usually pivot row & 1 other
	0 1 4 0 $\frac{1}{3}$ $\frac{1}{3}$ $8\frac{1}{3}$	A1	3	all correct (condone multiples of rows) Condone FT from one slip in (b)(i)
	0 0 2 1 $-\frac{2}{3}$ $\frac{1}{3}$ $\frac{1}{3}$			
	Max P now achieved	E1		Or "optimum", "P_{max} =...."etc" Bur must have no negatives in top row
	$P=51$	B1√		FT their tableau
	$x=8\frac{1}{3}, y=0, z=\frac{1}{3}$ (all three)	B1	3	correct values from almost 'correct' tableau (condone one slip) condone 8.33 or better
	Total		**15**	

JAN 2011

4(a)(i)	$\frac{4}{-1}=-4;\ \frac{10}{2}=5;\ \frac{21}{4}=5\frac{1}{4}$ 5 is smallest **positive** ratio	E1		Must see 5and $5\frac{1}{4}$ plus correct statement
	Pivot = 2	B1	2	
(ii)	$1\ \ 0\ \ -\frac{1}{2}\ \ 5\ \ 0\ \ \frac{3}{2}\ \ 0\ \ 15$	M1		row operations (even with wrong pivot)
	$0\ \ 0\ \ \frac{3}{2}\ \ 3\ \ 1\ \ \frac{1}{2}\ \ 0\ \ 9$	A1		1st, 2nd or last row correct
	$0\ \ 1\ \ \frac{1}{2}\ \ 2\ \ 0\ \ \frac{1}{2}\ \ 0\ \ 5$	A1		another of these correct
	$0\ \ 0\ \ 0\ \ -5\ \ 0\ \ -2\ \ 1\ \ 1$	A1		all correct (condone multiples of rows)
	Negative value in top row (→ optimum not reached)	E1	5	must have negative value in their top row
(b)(i)	New pivot is 'their $\frac{3}{2}$' in y-column PI	M1		or multiple of this
	$1\ \ 0\ \ 0\ \ 6\ \ \frac{1}{3}\ \ \frac{5}{3}\ \ 0\ \ 18$	A1		1st, 3rd or 4th row correct
	$0\ \ 0\ \ 1\ \ 2\ \ \frac{2}{3}\ \ \frac{1}{3}\ \ 0\ \ 6$	A1		another of these rows correct
	$0\ \ 1\ \ 0\ \ 1\ \ -\frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 2$			
	$0\ \ 0\ \ 0\ \ -5\ \ 0\ \ -2\ \ 1\ \ 1$	A1	4	all correct (condone multiples of rows)
(ii)	Optimum value of P reached	E1		must have no negative values in top row
	$P = 18$	B1√		ft their tableau
	$x=2, y=6, z=0$	B1√		$s=0, t=0, u=1$ (no more than 2 slips in final tableau for ft)
	$4x+2y+3z \le 21$ still has slack	B1	4	Tableau must indicate u is only slack variable

JUNE 2011

4(a)	$5x+3y+10z \leqslant 15$ $7x+6y+4z \leqslant 28$ $4x+3y+6z \leqslant 12$	M1 A1	 2	2 inequalities correct or all 3 LHS & RHS correct but using $<$ all correct
(b)(i)	Choosing 3 from bottom row as pivot	B1		identified or used
	$\begin{array}{cccccccc} 1 & 6 & 0 & 12-k & 0 & 0 & 2 & 24 \\ 0 & 1 & 0 & 4 & 1 & 0 & -1 & 3 \\ 0 & -1 & 0 & -8 & 0 & 1 & -2 & 4 \\ 0 & \frac{4}{3} & 1 & 2 & 0 & 0 & \frac{1}{3} & 4 \end{array}$	M1 A1 A1	 4	row operations (even with wrong pivot) one of rows 1, 2, 3 correct all correct (condone multiples of rows)
(ii)	$12-k<0$ $\Rightarrow k>12$	M1 A1	 2	their '$12-k$' <0 SC B1 for $k \geqslant 13$
(c)(i)	$\begin{array}{cccccccc} 1 & 6 & 0 & -8 & 0 & 0 & 2 & 24 \\ 0 & 1 & 0 & 4^* & 1 & 0 & -1 & 3 \\ 0 & -1 & 0 & -8 & 0 & 1 & -2 & 4 \\ 0 & \frac{4}{3} & 1 & 2 & 0 & 0 & \frac{1}{3} & 4 \end{array}$	M1		correct pivot from z column 4* (identified or used)
	$\begin{array}{cccccccc} 1 & 8 & 0 & 0 & 2 & 0 & 0 & 30 \\ 0 & \frac{1}{4} & 0 & 1 & \frac{1}{4} & 0 & -\frac{1}{4} & \frac{3}{4} \\ 0 & 1 & 0 & 0 & 2 & 1 & -4 & 10 \\ 0 & \frac{5}{6} & 1 & 0 & -\frac{1}{2} & 0 & \frac{5}{6} & \frac{5}{2} \end{array}$	A1 A1 A1	 4	one of rows 1, 3 or 4 correct another of rows 1, 3 or 4 correct all correct (condone multiples of rows)
(ii)	Maximum value of P now reached	E1		their tableau must have no negatives in top row
	$P=30,\ x=0,\ y=\frac{5}{2},\ z=\frac{3}{4}$	B1√		ft their values from their tableau provided at least 2 marks earned in (c)(i)
	$s=0,\ t=10,\ u=0$	B1cao	3	condone up to 2 slips in their final tableau
	Total		**15**	

JAN 2012

4(a)(i)	x-column pivot = 6 $\frac{2}{2}=1$, $\frac{3}{6}=\frac{1}{2}$ $\left(\text{and } \frac{1}{2}<1\right)$ } smallest **positive** quotient	B1 B1 E1	3	need to see correct quotients considered negative value **must** be mentioned as being considered but rejected
(ii)	$\begin{array}{cccccccc} \boldsymbol{P} & \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} & \boldsymbol{s} & \boldsymbol{t} & \boldsymbol{u} & \textbf{value} \\ 1 & 0 & 1 & 0 & 1 & \frac{1}{3} & 0 & 7 \\ 0 & 0 & 13 & 1 & 3 & -\frac{1}{3} & 0 & 1 \\ 0 & 1 & -5 & 0 & -1 & \frac{1}{6} & 0 & \frac{1}{2} \\ 0 & 0 & -14 & 0 & -4 & \frac{1}{6} & 1 & 4\frac{1}{2} \end{array}$	M1 A1 A1 A1	4	row operations 1st, 2nd or 4th row correct another of these 3 correct all correct (condone multiples of rows)
(b)(i)	No negatives in **top row**	E1	1	**but** must have no negative values in "their" top row
(ii)	One (inequality still has slack)	B1	1	
(c)(i)	$P=7$ $x=\frac{1}{2}$, $y=0$, $z=1$	B1√ B1 cao	2	FT their tableau condone one slip in final tableau
(ii)	Substituting "their" values from (c) (i) $\frac{1}{2}k+0+3=7$ $\Rightarrow k=8$	 M1 A1	2	
			13	

JUNE 2012

3(a)	P x y z s t value 1 $-k$ -6 -5 0 0 0			*may have 1's in 's' and 't' columns interchanged*
	0 2 1 4 1 0 11	B1		second row correct
	0 1 (3) 6 0 1 18	B1	2	third row correct
				may earn next B1 M1 if no slack variables
(b)	1 $2-k$ 0 7 0 2 36	B1		pivot is 3 (identified or used)
	0 $\frac{5}{3}$ 0 2 1 $-\frac{1}{3}$ 5	M1		row operations (even with wrong pivot) (obtaining 0 in pivot column)
	0 $\frac{1}{3}$ 1 2 0 $\frac{1}{3}$ 6	A1		first or second row correct
		A1	4	all correct (condone multiples of rows)
(c)(i)	($k=1 \Rightarrow$ max reached) since there are no negative values in top row	E1		provided there are no negative values in top row " all positive values... " scores E0
	(P_{max} =) 36	B1√	2	ft their tableau
(ii)	$k=3$: new pivot from x-column is $\frac{5}{3}$ used by attempting row operation	M1		ft their pivot if appropriate but must have slack variables
	1 0 0 $\frac{41}{5}$ $\frac{3}{5}$ $\frac{9}{5}$ 39 0 1 0 $\frac{6}{5}$ $\frac{3}{5}$ $-\frac{1}{5}$ 3 0 0 1 $\frac{8}{5}$ $-\frac{1}{5}$ $\frac{2}{5}$ 5	A1 √		first or last row correct ft one slip from their tableau in part (b) but must use correct pivot
		A1	3	all correct (condone multiples of rows)
	Optimum reached (or P_{max} = ...)	E1		must have earned M1 and have no negative values in top row
	39	B1√		ft their tableau
	$x=3,\ y=5,\ z=0$ $(s=0, t=0)$	B1 cso	3	must have correct final tableau
	Total		**14**	

JAN 2013

5(a)	P x y z r s t | value 1 −1 2 −3 0 0 0 | 0 0 1 1 1 1 0 0 | 16 0 1 −2 2 0 1 0 | 17 0 2 −1 2 0 0 1 | 19	B2,1,0	2	All correct, 3 rows correct
(b)(i)	z-col: $\frac{16}{1}, \frac{17}{2}, \frac{19}{2}$ Min, R_3 as pivot	M1 A1	2	
(ii)	1 $\frac{1}{2}$ −1 0 0 $1\frac{1}{2}$ 0 $\frac{51}{2}$ 0 $\frac{1}{2}$ 2 0 1 $-\frac{1}{2}$ 0 $\frac{15}{2}$ 0 $\frac{1}{2}$ −1 1 0 $\frac{1}{2}$ 0 $\frac{17}{2}$ 0 1 1 0 0 −1 1 2 **Alternative** 2 1 −2 0 0 3 0 | 51 0 1 4 0 2 −1 0 | 15 0 1 −2 2 0 1 0 | 17 0 1 1 0 0 −1 1 | 2	M1 A1 A1 (M1) (A1) (A1)	3	Row operations One row (other than R_3) correct All correct
(c)(i)	y col $\frac{15}{4}, \left(-\frac{17}{2}\right), \frac{2}{1}$ R_4 as pivot 1 $1\frac{1}{2}$ 0 0 0 $\frac{1}{2}$ 1 $\frac{55}{2}$ 0 $-1\frac{1}{2}$ 0 0 1 $1\frac{1}{2}$ −2 $\frac{7}{2}$ 0 $1\frac{1}{2}$ 0 1 0 $-\frac{1}{2}$ 1 $\frac{21}{2}$ 0 1 1 0 0 −1 1 2 **Alternative** 2 3 0 0 0 1 2 | 55 0 −3 0 0 2 3 −4 | 7 0 3 0 2 0 −1 2 | 21 0 1 1 0 0 −1 1 | 2	B1 M1 A1 (M1) (A1)	3	Fully correct description Row operations All correct
(c)(ii)	Optimal $P = \frac{55}{2}$ $x=0,\ y=2,\ z=\frac{21}{2}$ $s=t=0,\ r=\frac{7}{2}$	B1 B1 B1	3	Both statement and value needed. OE
		Total	**13**	

JUNE 2013

6(a)	P x y z r s t Value 1 −4 −3 −1 0 0 0 0 0 ② 1 1 1 0 0 25 0 1 2 1 0 1 0 40 0 1 1 2 0 0 1 30	B2,1,0	2	All correct, 3 rows correct
(b)	1 0 −1 1 2 0 0 50 0 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0 $\frac{25}{2}$ 0 0 $\frac{3}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ 1 0 $\frac{55}{2}$ 0 0 $\frac{1}{2}$ $\frac{3}{2}$ $-\frac{1}{2}$ 0 1 $\frac{35}{2}$	B1 M1 A1	 3	Pivot, x-col: 12.5, 40, 30 seen and correct pivot chosen Row operations All correct
(c)(i)	1 0 0 $\frac{4}{3}$ $\frac{5}{3}$ $\frac{2}{3}$ 0 $\frac{205}{3}$ 0 1 0 $\frac{1}{3}$ $\frac{2}{3}$ $-\frac{1}{3}$ 0 $\frac{10}{3}$ 0 0 1 $\frac{1}{3}$ $-\frac{1}{3}$ $\frac{2}{3}$ 0 $\frac{55}{3}$ 0 0 0 $\frac{4}{3}$ $-\frac{1}{3}$ $-\frac{1}{3}$ 1 $\frac{25}{3}$	B1 M1 A1	 3	Pivot, y-col: their 25, 55/3, 35 seen and correct pivot chosen Row operations All correct
(ii)	Max$P = \frac{205}{3}$ $x = \frac{10}{3}$, $y = \frac{55}{3}$, $z = 0$ $r = 0$, $s = 0$, $t = \frac{25}{3}$	B1 B1 B1ft	 3	Condone optimal, etc Ft on x and y All 3 must be stated

JUNE 2014

4(a)

P	x	y	z	r	t	
1	–3	–6	–2	0	0	0
0	1	3	2	1	0	11
0	3	4	2	0	1	21

B1 1st and 2nd row correct
B1 **2** 1st and 3rd row correct

(b)

P	x	y	z	r	t	
1	–1	0	2	2	0	22
0	1	3*	2	1	0	11
0	5	0	–2	–4	3	19

B1 Correct pivot 'y, 3' chosen and 11/3, 21/4 seen
M1 Row operations

Or

P	x	y	z	r	t	
1	–1	0	2	2	0	22
0	$\frac{1}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{11}{3}$
0	$\frac{5}{3}$	0	$-\frac{2}{3}$	$-\frac{4}{3}$	1	$\frac{19}{3}$

A1 **3** All correct

(c)

P	x	y	z	r	t	
5	0	0	8	6	3	129
0	0	15	12	9	–3	36
0	5*	0	-2	–4	3	19

B1 Correct pivot 'x, 5' chosen and 19/5, 11 seen
M1 Row operations

Oe

P	x	y	z	r	t	
1	0	0	$\frac{8}{5}$	$\frac{6}{5}$	$\frac{3}{5}$	$\frac{129}{5}$
0	0	1	$\frac{4}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	$\frac{12}{5}$
0	1	0	$-\frac{2}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	$\frac{19}{5}$

A1 **3** All correct

(d)

$P = 25.8$ B1
$z = r = t = 0$ B1
$x = 3.8, y = 2.4$ B1 **3**

Total **11**